# On the Multiple Access Interference in a DS/FFH Spread Spectrum Communication system

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## Abstract

This paper describes the influence of Multiple Access (MA) interference on the performance of a hybrid Direct Sequence Fast Frequency Hopping SSMA Communication system. MA-interference plays an important role in determining the total interference in a Spread Spectrum system, especially in a non-cellular environment where Power Control is hardly possible. A cross correlation factor which is directly responsible for this interference is derived. Using this factor a comparison between slow and fast frequency hopping (both in combination with direct sequence) is made. At the end two sets of Kasami codes are selected which offer a good performance.

## 1 Introduction

In recent years there has been much publications in the field of Spread Spectrum Communications, and more specific about Multiple-Access Interference ([1, 2] and others). In this paper we deal with a hybrid Direct Sequence, Fast Frequency Hop system in a non-cellular environment. Therefore the correlation function changes and also the performance analysis is due to changes.

The DS/FFH technique is an interesting technique for a number of reasons. In a non-cellular environment we have point-to-point communication without using a base station, this is a more flexible and cheaper method than the cellular approach. Because of the absence of base stations power control, playing a key role in reducing the Near-Far effect [3, 4], is no longer possible. Fast Frequency Hopping is therefore used to beat the Near-Far effect. On the other hand there is still Direct Sequence with its well known advantages: CDMA, Jamming rejection, fading rejection and Security. In [5] the FFH technique is also described, however in order to have a reasonable Processing Gain in a non-hybrid system the number of hop frequencies is large, the hit-chance becomes smaller and the MA-interference description can be simplified.

The organization of this paper is as follows: First in section 2 the hybrid system-model is described. Section 3 deals with the MA-interference, the measure for MAinterference in a DS/FFH system is derived, and a formula is given which gives the relation between this measure and the C/I-ratio. Section 4 deals with a comparison between DS/SFH and DS/FFH for Rayleigh fading channels in the absence of power control. Section 5 is about Pseudo Noise codes, in this section two sets of Kasami-codes are given which have good MA-interference properties. Finally section 5 gives some conclusions.

# 2 System Model

#### 2.1 What kind of a technique is DS/FFH?

The DS/FFH Spread Spectrum technique is a combination of direct-sequence and frequency-hopping. One data bit is divided over  $N_{FH}$  frequency-hop channels (carrier frequencies). In each frequency-hop channel one complete PN-code of length N is added to the data signal (see figure 1 where  $N_{FH}$  is taken to be 5, just for simplicity). Using the FFH scheme in stead of the SFH scheme causes the bandwidth to increase, this increase however is neglectable with regard to the enormous bandwidth already in use.



Figure 1: DS/FFH Spreading scheme

As the FH-sequence and the PN-codes are coupled, an address is a combination of an FH-sequence and  $N_{FH}$  PN-codes. To bound the hit-chance (the chance that two users share the same frequency channel in the same time) the frequency-hop sequences are chosen in such a way that two transmitters with different FH-sequences share at most two frequencies at the same time (timeshift is random).

Applying several frequency hops within each data bit, makes modulation by some kind of Phase Shift Keying problematic. An FSK (or MFSK) modulation scheme is therefore chosen.

#### 2.2 The DS/FFH SSMA system model

One of the important advantages of spread spectrum systems is their multiple access capability. The interference that results from simultaneous transmittance is called 'Multiple Access (MA) Interference'. Contribution to this interference occurs when the reference user and a nonreference user use the same FH-channel for a fraction of a frequency-hop.

Here we find the difference between DS and DS/FFH. In the case of DS all codes are transmitted in the same frequency slot, codes correlate in this case completely. In the DS/FFH case however two subsequent codes are transmitted in different frequency slots, so codes do only correlate partially. In figure 2 this is shown in more detail: In the DS-case  $code \ 1$  correlates with both  $code \ k$  and  $code \ k + 1$ , on the other hand in the DS/FFH case  $code \ 1$  only correlates with  $code \ k$ , the other codes are in other frequency slots.



Figure 2: DS versus DS/FFH

The DS/FFH SSMA system model that will be considered is shown below:



Figure 3: DS/FFH system model

Suppose there are K active users, one of them, user *i*, being the reference user. The other K - 1 users cause MA-interference. The data signal,  $b_k(t)$ , is a polar bit stream of unit amplitude, with a duration  $T_b$ . This signal is FSK-modulated. The FSK-spacing is  $\Delta_{FSK}$ .  $a_k(t)$  is the waveform of the PN-code, also a sequence of polar bits with unit amplitude, with duration  $T_c$ . If we denote by  $d_k(t)$  the sequence  $\{-\frac{N_{FH}}{2} + \frac{1}{2}, -\frac{N_{FH}}{2} + 1\frac{1}{2}, \cdots, \frac{N_{FH}}{2} - \frac{1}{2}\}$  of duration  $T_h = NT_c = T_b/N_{FH}$ , the transmitted signal of the k-th user is:

$$s_{k}(t) = \sqrt{2Psin\{(\omega_{c} + d_{k}(t)\Delta_{FH} + \frac{1}{2}b_{k}(t)\Delta_{FSK})t + \frac{\pi}{2}a_{k}(t) + \theta_{k}\}}$$

$$= \sqrt{2P}a_{k}(t)cos\{(\omega_{c} + d_{k}(t)\Delta_{FH} + \frac{1}{2}b_{k}(t)\Delta_{FSK})t + \theta_{k}\}}$$

$$(1)$$

 $\Delta_{FH}$  is the frequency-hop spacing. *P* is the common transmitted power,  $\omega_c$  the center-frequency and  $\theta_k$  the phase of the *k*-th carrier.

For the time being we assume that collision between the k-th and the i-th user in one frequency-hop channel always occurs. Later this can be corrected by evaluating the hit chance [6]. The FH-spacing is chosen so that double frequency terms can be removed after modulation  $(\Delta_{FH} \gg T_h^{-1})$ .

The received signal is now:

$$r(t) = \sum_{\substack{k=1\\d_k(t)\Delta_{FH} + \frac{1}{2}b_k(t)\Delta_{FSK}(t)}^K K(t) \Delta_{FSK}(t) + n(t)$$
(2)

A correlation receiver, synchronized with user i, responds to r(t) with an output

$$Z_{i} = \sqrt{2P} \{T_{h} + \sum_{k=1, k \neq i}^{K} R_{k,i}(\tau_{k}) \cdot \cos\phi_{k} \} + \int_{0}^{T_{h}} n(t)a_{i}(t)\cos(\cdots)dt$$

$$(3)$$

Here  $\phi_k = \theta_k + \omega_c \tau_k$ ,  $\tau_k$  is the delay of the k-th user ( $\tau_k$  is defined to be positive if user i is delayed with regard to user k). So  $\tau_k$  can vary between  $-T_h$  and  $T_h$ , n(t) is the channel noise,  $cos(\cdots)$  denotes the same cos-term as in (2) and  $R_{k,i}(\tau_k)$  is a continuous time partial cross correlation function defined by:

$$R_{k,i}(\tau_k) = \int_{\tau_{k1}}^{\tau_{k2}} a_k(t+\tau_k) a_i(t) dt$$
 (4)

In the DS-case there are two correlation terms [1]. The interval  $(\tau_{k1}, \tau_{k2})$  is the time that two users *i* and *k* share the same frequency-hop channel.  $\tau_{k1}$  and  $\tau_{k2}$  are related to  $\tau_k$  in the following way:

$$\tau_{k1} = \begin{cases} 0, & \tau_k \leq 0\\ \tau_k, & \tau_k > 0 \end{cases}$$
$$\tau_{k2} = \begin{cases} \tau_k + T_h, & \tau_k \leq 0\\ T_h, & \tau_k > 0 \end{cases}$$

If  $-T_h \leq lT_c \leq \tau_k \leq (l+1)T_c \leq T_h$ ,  $(l \in \{\ldots -2, -1, 0, 1, 2, \ldots\})$  this correlation function can be written as:

$$R_{k,i}(\tau_k) = C_{k,i}(l) \{ (l+1)T_c - \tau_k \} + C_{k,i}(l+1)(\tau_k - lT_C)$$
(5)

 $C_{k,i}(l)$  is the discrete aperiodic cross correlation function [7]:

$$C_{k,i}(l) = \begin{cases} \sum_{\substack{j=0\\ N-1+l}}^{N-1-l} a_j^{(k)} a_{j+l}^{(i)}, & 0 \le l \le N-1\\ \sum_{\substack{j=0\\ j=0}}^{N-1+l} a_{j-l}^{(k)} a_j^{(i)}, & 1-N \le l < 0\\ 0, & |l| \ge N \end{cases}$$

#### 3 MA-interference parameter

In this section an expression for the measure of the MAinterference is derived. We use results from [1, 8] extended to a DS/FFH SSMA system.

In this approach phase shifts and time delays are treated as independent random variables. As we are only interested in choosing a good set of PN-codes, the only interference considered is MA-interference.  $\phi_i$  and  $\tau_i$  are assumed to be 0.

The desired signal component of  $Z_i$  is now  $\sqrt{P/2}T_h$  while the variance of the MA-component of  $Z_i$  is (under Gaussian assumption):

$$Var\{Z_{i,MA}\} = \frac{P}{8T_h} \sum_{k=1,k\neq i}^{K} \int_{-T_h}^{T_h} R_{k,i}^2(\tau) d\tau$$
$$= \frac{P}{2T_h} \sum_{k=1,k\neq i}^{K} \sum_{l=1-N}^{N-1} \int_{lT_c}^{(l+1)T_c} R_{k,i}^2(\tau) d\tau + \frac{N_0 T_h}{4}$$
(6)

As  $\tau$  can take values between  $-T_c$  and  $T_c$ ,  $\tau$  is integrated over the corresponding interval. Substituting  $R_{k,i}(\tau)$  from (5) and evaluating the integral gives:

$$Var\{Z_{i,MA}\} = \frac{PT_{h}^{2}}{24N^{3}} \left\{ \sum_{k=1,k\neq i}^{K} \rho_{k,i} \right\} + \frac{N_{0}T_{h}}{4} \quad (7)$$

where:

$$\rho_{k,i} = \sum_{l=1-N}^{N-1} \left\{ C_{k,i}^2(l) + C_{k,i}(l)C_{k,i}(l+1) + C_{k,i}^2(l+1) \right\}$$

The term  $\rho_{k,i}$  is directly responsible for the Multiple-Access interference with the *k*-th user. After some calculation we find for the mean and variance:

$$E[\rho_{k,i}] = 2N^2 - 1 \tag{8}$$

$$Var[\rho_{k,i}] = 6N^3 - 5N^2 + 2N \tag{9}$$

Interesting to mention is the relation between this factor and the  $r_{k,i}$ -factor from [1]:  $\rho_{k,i} = r_{k,i} - 1$ . The  $\rho_{k,i}$ term is directly responsible for the MA-interference in a DS/FFH system, on the basis of this term proper PN-codes can be selected.

Assuming the mean value for  $\rho_{k,i}$ , the resulting signal to interference ratio (S/I) becomes:

$$S/I_{FFH} = \frac{2E_{b,i}/N_0}{1 + \frac{1}{6N} \cdot \frac{2}{N_0} \sum_{k=0, k \neq i}^{K} E_{b,k}}$$
(10)

Here  $E_{b,k}$  is the received energy per bit for the  $k^{th}$  transmitter,  $N_0$  is the single-sided spectral noise density, and K is the number of users which share together with the reference user the same frequency hop channel at the same time (they have a hit).

In this formula we did not exploit the diversity which is inherent to fast frequency hopping. As only one bit is transmitted in  $N_{FH}$  frequency slots, it is possible to detect all frequency hops separately and then make a majority vote, applying this strategy yields an enhanced performance.

When applying majority voting the signal to interference ratio (per frequency hop) changes:

$$S/I_{FFH,MV} = \frac{2E_{h,i}/N_0}{1 + \frac{1}{6N} \cdot \frac{2}{N_0} \sum_{k=0,k\neq i}^{K} E_{b,k}}$$
(11)

Here  $E_h$  is the energy per frequency hop.

One last remark is to be made. In the derivation of the signal to interference ratio the signals are assumed to be Gaussian, this assumption is only valid for either long PN-codes or a large number of users. Simulations show that for a FH-pattern with maximal two hits and a PN-code length of 63, 2 interfering users are enough to have Gaussian distributed interference.

#### 4 FFH versus SFH

In this section we will compare the performance of DS/FFH systems and DS/SFH systems. As this section is only meant as an illustration, we will not go into detail.

For  $\rho_{k,i}$  a value of  $2N^2$  is chosen  $(E[\rho_{k,i}] = 2N^2 - 1)$ , in section 5 will be shown that sets of PN-codes can be found which have  $2N^2$  as an upper bound. For  $N_{FH}$  we chose a value of 7 (Now there are 6 FH-sequences which share maximally two frequencies at the same time for a random time shift: 0-1-2-3-4-5-6, 0-2-4-6-1-3-5, 0-3-6-2-5-1-4, 0-4-1-5-2-6-3, 0-5-3-1-6-4-2, 0-6-5-4-3-2-1). and the PN-code length is 63. For both SFH (see [9] for details) and FFH it is assumed that there are  $N_{FH}$  frequency slots available.

The hit chance  $(P_{hit})$  is the chance that two transmitters share the same frequency slot at the same time. For FFH there is a maximum chance of 1 that there appear 2 hits at a random time-shift (there is only 1 hit if the time-shift is an integer number times a hop-time, this is explained in [10] there they don't mention the 'integer'-condition). So for the FFH case there are maximally  $2(K - 1)/N_{FH}$  users transmitting in the same frequency-slot as the reference user, K being the number of active users.

For SFH the approximation found in [9] will be used, which results in:  $P_{hit, SFH} = 1/(N_{FH} + 1) = 0.14$ . For the S/I-ratio the formula by Pursley will be used [1] (The  $r_{k,i}$ -factor will also be approximated by its mean:  $2N^2$ ):

$$S/I_{SFH} = \frac{2E_{b,i}/N_0}{1 + \frac{1}{3N} \cdot \frac{2}{N_0} \sum_{k=0,k \neq i}^{K} E_{b,k}}$$
(12)

As no power control is assumed we have to make an assumption concerning the spatial distribution of the users. For this we will use the log-normal distribution function motivated in [11]:

$$K'(r) = \frac{K - 1}{\sqrt{2\pi} r s_i} exp\left[-\frac{(lnr)^2}{2s_i^2}\right]$$
(13)

Here K'(r) is the number of interfering users per unit distance at a normalized distance r.  $s_i$  is a parameter which determines the shape of the distribution this parameter is chosen to have a value of 0.7.

The power is expected do decrease as a function of the normalized distance:  $P(r) \sim r^{-\beta}$ , for  $\beta$  a value 3 has been chosen. The reference user is assumed to have a normalized distance r = 0.7 to the receiver. The resulting spatial distribution for 50 users is shown in figure 4.

The channel is assumed to have Rayleigh fading, so the relation between the S/I-ratio ( $\gamma$ ) and the Bit Error Rate for the FSK modulation scheme is given by [12]:

$$BER = \frac{1}{2+\gamma} \tag{14}$$

When comparing DS/FFH and DS/SFH without applying the diversity inherent to FFH both systems will perform about the same (see figure 5). More interesting is the case



Figure 4: Spatial distribution of users

in which we apply majority voting (diversity), the results are given in figure 6 for 1, 11, 31 and 51 users. For the majority voting it is assumed that the interference of two successive 'hops' is independent (different frequency channels, pseudo-random hop-sequences), this is not true in the SFH case.

# 5 Finding a proper set of PN-codes

In the previous section a measure was derived for the MAinterference in a DS/FFH SSC system. This measure will function as a criterion to select a proper set of PN-codes. Except for a low cross-correlation factor the codes also have to meet the "balance"-property: the difference between ones and zeros in the code may only be 1. This last requirement stands for nice spectral density properties. Candidates are codes from the large set of Kasami codes, the code-length is chosen to be 63. In [13] also code-set selections can be found, candidates here are the Gold codes and the small set of Kasami codes, the "balance"-criterion is not met in that case.

Kasami codes in the large set are created by combination of 3 m-sequences [14, 8, 15]:

$$r = u \cdot T^k v \cdot T^m w$$

Here u and v are m-sequences of length:  $N = 2^n - 1$  (n even) which form a preffered pair [16]. w is a m-sequence resulting after decimation the v-code with a value  $2^{n/2} + 1$ . T denotes a delay of one element, k is the offset of the v-code with respect to the u-code and m is the offset of the w-code with respect to the u-code. Offsets are relative to the all-ones state.

The following notation will be used for Kasami-codes of the large set with n = 6 (N = 63), u is the m-sequence with feedbacks (6,1) and v is the m-sequence with feedbacks (6,5,2,1):



Figure 5: BER as a function of the S/N-ratio for both SFH and FFH, no majority voting applied



Figure 6: BER as a function of the S/N-ratio for FFH when majority voting is applied

$$kas(n,k,m) = \begin{cases} u \cdot T^{k}v \cdot T^{m}w, \\ 0 \le k \le 2^{n} - 2, 0 \le m \le 2^{n/2} - 2 \\ u \cdot T^{m}w, (small \ set) \\ k = 2^{n} - 1, 0 \le m \le 2^{n/2} - 2 \\ v \cdot T^{m}w, \\ k = 2^{n}, 0 \le m \le 2^{n/2} - 2 \\ u \cdot T^{k}v, (Gold \ codes) \\ 0 \le k \le 2^{n} - 2, m = 2^{n/2} - 1 \\ v, (m - sequence) \\ k = 2^{n} - 1, m = 2^{n/2} - 1 \\ u, (m - sequence) \\ k = 2^{n}, m = 2^{n/2} - 1 \end{cases}$$

To calculate the total MA-interference in a SSC system, we have to know the  $\rho_{k,i}$ -factor of each code combination used. The mean-value of this term is  $2N^2 - 1$ , as a selection-criterium we require that for all code combinations in the set the  $\rho_{k,i}$ -factor has a value less than this mean:

$$\rho_{k,i} < 2N^2 - 1$$

Let us look at a set of PN-codes with  $\rho_{k,i}$ -factors below  $2N^2 - 1$ . The PN-codes in this example have length 63, so there are 520 possible PN-codes [8]. It appeared that among these 520 codes only 241 codes are balanced, these 241 balanced codes are taken as a candidate set.

It turned out that there are two sets, each containing 17 PNcodes, for which all  $\rho_{k,i}$ -factors have values below  $2N^2$ . These sets are shown in the tables 1 and 2.

To determine the number of addresses which can be formed with these sets of codes, remember that if there are  $N_{FH}$  + 1 FH-channels available,  $N_{FH}$  FH-sequences can be formed. Another constraint is that one user address uses  $N_{FH}$  different PN-codes (to avoid the case that two addresses have the frequency-hop sequence as well as more than one PN-code in common). When  $N_c = 17$  (number of codes in a code set) the resulting number of addresses is then about  $8 \cdot 10^6$ .

Table 1: set 1 of PN-codes with  $\rho_{k,i} < 2N^2$  with N = 63

kas(6,46,3)	kas(6,50,0)	kas(6,57,0)
kas(6,60,0)	kas(6,14,1)	kas(6,21,1)
kas(6,26,1)	kas(6,44,1)	kas(6,51,1)
kas(6,8,3)	kas(6,16,3)	kas(6,1,4)
kas(6,17,4)	kas(6,30,4)	kas(6,3,5)
kas(6,31,5)	kas(6,64,7)	

Table 2: set 2 of PN-codes with  $\rho_{k,i} < 2N^2$  with N=63

kas(6,50,0)	kas(6,60,0)	kas(6,26,1)
kas(6,44,1)	kas(6,51,1)	kas(6,53,3)
kas(6,38,4)	kas(6,44,4)	kas(6,3,5)
kas(6,11,5)	kas(6,31,5)	kas(6,32,5)
kas(6,41,5)	kas(6,62,5)	kas(6,0,6)
kas(6,18,6)	kas(6,64,7)	

# 6 Conclusions

In order to reduce the MA-interference in a DS/FFH SSMA communication system a criterion for choosing sets of PN-codes was derived. First a factor  $(\rho_{k,i})$  is derived which is directly responsible for the MA-interference in DS/FFH SSMA systems. Comparing this factor with a corresponding factor in a DS/SFH or DS SSC system [6, 1] it appears that, if no diversity is applied, DS/FFH systems can have have about the same performance . However when applying majority voting, making use of the fact that two successive hops are independent, the DS/FFH system performs much better. This is illustrated by BER versus  $E_b/N_0$  plots for both techniques.

In most applications a lot of users have to be allowed in the system. In such a case the Bit Error Rate is almost totally determined by the MA-interference. This makes sets of PN-codes with good cross-correlation properties attractive. The sets presented in section 5 are the proper sets to use under these circumstances.

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